UE Fluids F15 Solution Fall '07

1) Easiest to do in polar coordinates: $\psi = V_\infty \sin\theta \left(r - \frac{R^2}{r}\right) + 1$

$$\frac{\partial \psi}{\partial r} = V_\infty \sin\theta \left(1 + \frac{R^2}{r^2}\right), \quad r\frac{\partial \psi}{\partial r} = V_\infty \sin\theta \left(r + \frac{R^2}{r}\right)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) = \frac{1}{r}\left[V_\infty \sin\theta\left(1 - \frac{R^2}{r^2}\right)\right] = V_\infty \sin\theta\left(\frac{1}{r} - \frac{R^2}{r^3}\right)$$

$$\frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = \frac{1}{r^2}\left[-V_\infty \sin\theta\left(r - \frac{R^2}{r}\right)\right] = -V_\infty \sin\theta\left(\frac{1}{r} - \frac{R^2}{r^3}\right)$$

cancel

$$\nabla^2 \psi \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = 0 \quad \checkmark$$

2) At $r = R$, $\psi = V_\infty \sin\theta\left(R - \frac{R^2}{R}\right) = 0 = \text{constant}$ ✓

3) Easiest to do in cartesian coordinates: $\psi = V_\infty\left(y - \frac{yR^2}{x^2+y^2}\right) + 1$

$$u = \frac{\partial \psi}{\partial y} = V_\infty\left[1 - \frac{R^2}{x^2+y^2} + \frac{yR^2}{(x^2+y^2)^2}\cdot 2y\right] = V_\infty\left[1 - \frac{R^2}{r^2} + 2\frac{R^2\sin^2\theta}{r^2}\right]$$

"At infinity" means $r \to \infty$ or $\sqrt{x^2+y^2} \to \infty$

$$\lim_{r\to\infty} u = V_\infty\left[1 - \cancel{\frac{R^2}{r^2}} + 2\cancel{\frac{R^2\sin^2\theta}{r^2}}\right] = V_\infty \quad \checkmark$$

UE Fluids, F16 Solution Fall '07

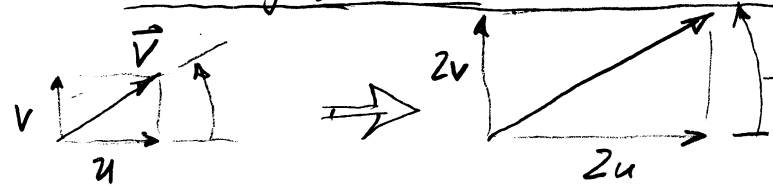
1) Anderson 3.16

Nonlifting flow over circular cylinder: $\psi = V_\infty\left(y - \frac{yR^2}{x^2+y^2}\right) = V_\infty \sin\theta\left(r - \frac{R^2}{r}\right)$

If we double $V_\infty = 20 \rightarrow 40$ ft/s, ψ will simply double everywhere, $\psi \rightarrow 2\psi$

The velocity vectors $(u, v) = \left(\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}\right)$ will also double, $(u, v) \rightarrow (2u, 2v)$

But the <u>velocity direction will not change</u>. $\arctan\frac{v}{u} \rightarrow \arctan\frac{2v}{2u}$ same



If $\vec{V}$ has the same direction, then streamline pattern is the same.

2) Anderson 3.18

$L' = \rho V_\infty \Gamma = 6$ N/m (given) $\qquad$ N/m $= \frac{kg\,m}{s^2}\frac{1}{m} = \frac{kg}{s^2}$

Also, we have $V_\infty = 30$ m/s, and $\rho = 1.226$ kg/m^3 (sea level)

$$\boxed{\therefore\ \Gamma = \frac{L'}{\rho V_\infty} = \frac{6\ kg/s^2}{1.226\ kg/m^3 \cdot 30\,m/s} = 0.163\ m^2/s}$$